B.Sc. Semester-IV Examination, 2022-23

MATHEMATICS [Honours]

Course ID : 42112 **Course Code : SH/MTH/402/C-9**

Course Title : Multivariate Calculus

Time : 2 Hours Full Marks : 40

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meaning.

UNIT-I

1. Answer any **five** from the following questions: 2×5=10

a) If $f(x,y)=\frac{1}{|x|}+\frac{1}{|y|}, x\neq 0$ and $y\neq 0$ then show that $f(x,y)\to\infty$ as $(x,y)\to(0,0)$.

b) Let $f(x,y)=\begin{cases}\frac{xy}{x^2+y^2} & if\ (x,y)\neq(0,0)\\ 0 & if\ (x,y)=(0,0)\end{cases}$. Show that the directional derivative does not exist for the function f in any arbitrary direction at the poit (0, 0).

c) Evaluate $\int_0^{\pi}\int_0^{\cos\theta} r\sin\theta\, dr d\theta$.

[Turn Over]

d) Show that $z=f(x^2y)$, where f is differentiable, satisfies $x\left(\frac{\partial z}{\partial x}\right)=2y\left(\frac{\partial z}{\partial y}\right)$.

e) Integrate $\iint_D xy\, dx\, dy$, where D is the domain bounded by the x-axis, ordinate $x=2a$ and the curve $x^2 = 4ay$.

f) If $\vec{f}=t^2\hat{i}+t\hat{j}-t^3\hat{k}$ and $\vec{g}=\sin t\hat{i}-\cos t\hat{j}-t^3\hat{k}$ then find $\frac{d}{dt}(\vec{f}\times\vec{g})$.

g) If V be the region enclosed by the surface S and $\hat{n}$ be the unit outward drawn normal to it, then prove that $\iint_S \hat{n}\, dS=\vec{0}$.

h) Find the curl of $\vec{f}(x,y,z)=(x+y+1)\hat{i}+\hat{j}-(x+y)\hat{k}$, then prove that $\vec{f}.curl\,\vec{f}=0$.

UNIT-II

2. Answer any **four** from the following questions: 5×4=20

a) State the necessary and sufficient conditions for the existence of maximum or minimum of a function of two variables and hence show that the minimum value of $u(x,y)=xy+c^3\left(\frac{1}{x}+\frac{1}{y}\right)$ is $3c^2$.

b) Show that the function $f(x,y)=\sqrt{|xy|}$ is not differentiable at the point (0, 0).

c) Evaluate $\iint_R |cos(x+y)|dxdy$ where $R=\{(x,y)\,|\,0\le x\le\pi, 0\le y\le\pi\}$.

d) If $\vec{f}=2y\hat{i}-z\hat{j}+x^2\hat{k}$ and S is the surface of the parabolic cylinder $y^2 = 8x$ of the first octant bounded by the planes $y = 4$, $z = 6$, then evaluate surface integral $\iint_s \vec{f}.\hat{n}\, dS$.

e) Evaluate $\iint_S (\vec{\nabla}\times\vec{A}).\hat{n}dS$ where $\vec{A}=(x-z)\hat{i}+(x^3+yz)\hat{j}-3xy^2\hat{k}$ and S is the surface of the cone $z=2-\sqrt{x^2+y^2}$ above the xy plane.

f) Show that the vector $\vec{F}=(2x-yz)\hat{i}+(2y-xz)\hat{j}+(2z-xy)\hat{k}$ is irrotational. Hence find a scalar point function φ, such that $\vec{F}=\vec{\nabla}\varphi$.

UNIT-III

3. Answer any **one** of the following questions:

$10\times1=10$

a) i) If $y=F(x,t)$, where F is a differentiable function of two independent variables x and t which are related to two variables u, v by the relations:

$u=x+ct, v=x-ct.$

then prove that the partial differential equation $\frac{\partial^2 y}{\partial x^2}-\frac{1}{c^2}\frac{\partial^2 y}{\partial t^2}=0$ can be transformed into $\frac{\partial^2 y}{\partial u\partial v}=0$. 5

ii) Evaluate $\oint_C (xy\,dx+xy^2dy)$ by Green's theorem, where C is the square in the xy plane with vertices (1, 0), (0, 1), (–1, 0) and (0, –1). 5

b) i) Define consevative vector field. If $\vec{F}=\vec{\nabla}\varphi$ is a single valued function and has continuous partial derivatives, then prove that the work done in a moving particle form one point to other in this field is independent of the path joining the two points.

ii) Find the maximum or minimum value of $f(x,y,z)=x^m y^n z^p$ subject to the condition $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=1$ using the method of Lagranges multipliers. 5+5

———